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- an equivalent energy approach -

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Shot impact vibration damper
– an equivalent energy approach –

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Appendix (Nomenclature)

A_r – displacement amplitude of resonance vibration
$A_r = A(\mu=0)$ – without shot damper
$A(\mu \neq 0)$ – with shot damper
c – damping or friction coefficient (in dependence of index)
$D = \dfrac{d}{A_r}$ – dimensionless clearance
d – clearance of shot damper
E – efficiency of vibration reduction
E_v – kinetic energy
E_d – dissipated energy
F_s – dry friction force of the shot bag
F_0 – forcing amplitude
K – stiffness in the main system
m – mass of the shot or shot bag
M – mass of the main system
$\mu = \dfrac{m}{M}$ – dimensionless shot damper mass
R – impact restitution coefficient
T – vibration period
$\omega = 2\pi f$ circular frequency
η – system loss factor
$\psi = 2\pi\eta$ system damping capacity
x – main system coordinate
y – shot center of mass coordinate

1. Introduction

The current design trends of structures and machinery are characterized by, for example, mass optimization and a working capacity increase. This usually means that higher working loads are acting on less stiff and lighter mechanical structures; in consequence higher vibration amplitudes occur, and these are inconvenient for many reasons.

The classical cure is detuning the structure resonances by the modification of its mass or its stiffness, adding additional damping or installing additional mass subsystems called vibration dampers and absorbers.

Among the many design concepts of vibration absorbers, the multi-unit impact damper based on granular material (shot) is now emerging as a concurrent design concept for applications.

The history of the impact dampers begins with works by Paget [1], Kobrinsky [2], Masri [3], Cempel [4], Bapat and Popplewell [5] and others. The existing theory of multi-unit dampers may be divided into two approaches: the analytical one, developed by Masri and later on by Bapat and Sankar [7], and the equivalent continuous force approach elaborated by Cempel [6]. As regards the shot dampers (very many particles), various results of experimental work have been published by Popplewell [8],[9], Arnold [10], and only one analytical approach by Araki [11] combined with experimental verification. The experimental work pinpoints the conclusion that it is worthwhile applying shot dampers. However, the existing analytical approach does not give a simple design rule for the shot damper. This design rule has to give indications with respect to the choice of the parameters of the shot damper. These parameters are the mass of the shot damper and container clearance (see Fig. 1).

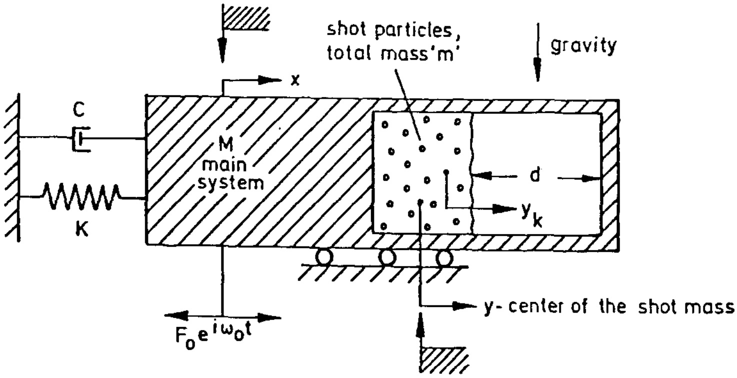

Fig. 1:
Sketch of the physical model of a shot damper

As these papers show, they may be determined starting with the known characteristics of structure vibration; the reduced mass (M), loss factor (η) and the desired rate of amplitude reduction (E) at the given point of the structure.

The presentation of such a design rule is the main aim of the paper. The equivalent energy approach is taken as a tool for theoretical consideration here. The theoretical results were partially verified by experiment using a laboratory model.

2. The model of a shot damper

Let us assume that the vibrating structure is linear and has well-separated resonances. At the frequency of interest, where resonant vibration needs reduction, it can be presented as a one degree of freedom model with parameters $M-K-C$, forced harmonically at the resonant frequency ω_0, (see Fig. 1). Assuming, further, that at the point of high vibration amplitude a container is attached carrying an amount of shot (lead, steel or other granular material) of a few millimetres diameter and total mass (m). This container is not filled up, and there is some clearance (d), which is simply the linear dimension of the remaining free way in the direction of motion (see Fig. 1). We assume, further, that the direction of motion is perpendicular to the gravity force, so it does not interact with the inertia forces of the shot material.

The shot itself can be loose, or filled loosely in bags of some kind of elastic foil, mesh etc. The latter case reveals the clearance (d) quite well and also greatly improves the shot damper performance, as was found by Popplewell et al. [8], [9].

We assume, further, that during the motion, each particle, or group of particles, or the whole of the shot bag, strike the container walls twice. There may also be frictional interaction between shot particles and container as well as between the particles themselves.

The description of motion of such a system is not an easy task. A strict analytical approach prefers the description of each particle motion and impact modelling by Dirac delta functions [4], [6] or other discontinuity [7]. A phenomenological approach is also possible, based on some observations concerning the motion of the shot as a whole or the energy exchange between the shot and the container. The equivalent continuous force approach can be applied also to multi-unit dampers as developed earlier by the first author [6]. The method adopted in this paper will be based on the equivalent mass approach to energy exchange in the system. It describes the motion of all the shot in an average. So the coordinates, velocity and acceleration, if needed, will concern the centre of the mass of the shot bag or bags or even loose shot.

3. The energy approach applied to the shot damper

Consider two kinds of energy in the system in Fig. 1. Kinetic, vibration energy is at a maximum just before the impact, since denoting the velocities as $\dot{x} = \dot{x}_i$, $\dot{y} = \dot{y}_i$, we obtain

$$E_v = \frac{1}{2} M \dot{x}_i^2. \tag{1}$$

The energy dissipated by the impact of shot particles or bag or one bag only, together with frictional dissipation during shot motion one can find on the basis of [12]:

$$E_d = E_{d.imp.} + E_{d.frict.}$$
$$E_d = \frac{1\text{-}R^2}{2} \frac{mM}{m+M} (\dot{x}_i - \dot{y}_i)^2 + E_{d.frict.} \tag{2}$$

Then the damping capacity introduced to the main (M, K, C) system as a ratio of both energies will be

$$\psi_s = \frac{E_d}{E_v} = \frac{\frac{1-R^2}{2}\,\frac{m}{1+\mu}\,(\dot{x}_i - \dot{y}_i)^2 + E_{d.frict.}}{\frac{1}{2} M \dot{x}_i^2}, \tag{3}$$

here

$$\mu = \frac{m}{M},$$

and R is the coefficient of velocity restitution during the impact.

The second part of the dissipation during dry friction will be calculated later on. Looking now for the maximum value of the damping capacity we need:

- purely plastic impact with i.e. $R \cong 0$
- the maximum relative velocity of the striking masses $\dot{x}_i = -\dot{y}_i$.

Taking this assumption and knowing that the absorber mass ratio μ is of maximum order a few per cent $\mu \ll 1$, instead of (3) one can obtain that the damping capacity in the main system due to impacts only is

$$\psi_i \cong 4\mu. \tag{4}$$

The damping capacity is not well known, we generally use the loss factor (η) and therefore $\psi = 2\pi\eta$. So the loss factor due to impacts in the system will be

$$\eta_i = \frac{\psi_i}{2\pi} = \frac{2}{\pi}\mu \cong 0.64\,\mu. \tag{5}$$

This means that operation of the shot damper *introduces to the system the loss factor of the order of one half of the damper-reduced mass.*

It is interesting to quote here the paper by Araki [11] (3-rd Report), where he found

$$\eta_{i,hor} = (0.6 \div 0.8)\,\mu$$

for horizontally working systems

and $\eta_{j,vert} \cong 0.4\,\mu$

for vertically working systems.

This seems to be good confirmation of the rough consideration presented above.

Calculation of the system loss factor due to friction will be done now. Assume that the shot friction force can be expressed as

$$F_s = \mathrm{sgn}\,(\dot{x} - \dot{y}) \cdot c_f. \tag{6}$$

By means of equivalent linearization we can express it approximately [13]

$$F_s \approx c_{eq}\,(\dot{x} - \dot{y}) \tag{7}$$

where

$$c_{eq} = \frac{4c_f}{\pi\omega_0\,|x-y|}\,;\ |x-y| = \text{relative shot displacement.}$$

The equivalent coefficient of viscous damping can then be transformed as a loss factor for shot only

$$\begin{aligned}\eta_m &= \frac{c_{eq}}{m\omega_0} = \frac{4c_f}{\pi m\omega_0^2\,|x-y|} = \frac{4F_s}{\pi m\omega_0^2\,|x-y|} = \\ &= \frac{4}{\pi}\,\alpha_f, \quad \alpha_f = \frac{F_s}{m\omega_0^2\,|x-y|}.\end{aligned} \tag{8}$$

Finally, for the whole system loss factor we have

$$\eta_f = \frac{c_{eq}}{M\omega_0} = \frac{c_{eq}m}{M\omega_0 m} = \frac{4\mu}{\pi}\,\alpha_f. \tag{9}$$

So in both cases the loss factor depends on a coefficient which is, as one can see from (8), a ratio of the friction force F_s to the relative inertia force of the shot mass (if motion is assumed to be harmonic) i.e. $m\omega_0^2|x-y|$. For the safe and stable operation of the shot damper this coefficient has to be less than one, $0<\alpha_f<1$ and we may keep it as almost optimal when

$$\alpha_f \cong 0.5. \tag{10}$$

Returning not to the definition of the damping capacity and system loss factor we may thus express the externally introduced damping by shot as

$$\begin{aligned}\eta_e &= \eta_i + \eta_f \\ &= \frac{2}{\pi}\,\mu(1+2\,\alpha_f) = \begin{cases} \dfrac{2}{\pi}\,\mu - \text{no friction } (\alpha_f=0) \\[2ex] \dfrac{4}{\pi}\,\mu - \text{optimal shot friction } (\alpha_f=0.5). \end{cases}\end{aligned} \tag{11}$$

We have just calculated the damping introduced to the main system by the shot damper. But, as follows from Fig. 1, the system itself has damping

$$\eta = \frac{C}{M\omega_0}$$

which is responsible for the resonance amplitude of the main system M, K, C (see Fig. 1)

$$A_r(\mu=0) = \frac{x_{st}}{\eta}, \quad x_{st} = \frac{F_0}{K}. \tag{12}$$

Hence the total loss factor η_s in the main system and its resonant amplitude will be

$$\eta_s = \eta + \eta_e = \eta + \frac{2}{\pi}\,\mu(1+2\,\alpha_f),$$

$$A_r(\mu \neq 0) = \frac{x_{st}}{\eta_s} = x_{st}[\eta + \frac{2\mu}{\pi}(1+2\alpha_f)]^{-1}. \tag{13}$$

From the applicational point of view the efficiency of the damper mounting is important. According to definition [13] it is expressed as a ratio of amplitude without damper to the amplitude of the modified system with damper. So from (12) and (13) we have

$$E \equiv \frac{A_r(\mu=0)}{A_r(\mu\neq 0)} = 1 + \frac{\eta_e}{\eta}$$

$$= 1 + \frac{2}{\pi}\frac{\mu}{\eta}(1+2\alpha_f) = \begin{cases} 1+0.64\,\frac{\mu}{\eta} & \text{– no friction of the shot} \\ 1+1.28\,\frac{\mu}{\eta} & \text{– optimal shot friction.} \end{cases} \tag{14}$$

Hence one can see that by choosing the optimal friction of the shot inside the container one may increase the efficiency of a shot damper twice. With respect to the question of how to fill the container, i.e. with loose or packed shot, one may recall that plastic impacts were assumed, $R = 0$. Hence the only possible solution is to use small plastic bags of shot, because in this case the impact is almost purely plastic giving no rebound. But this last conclusion depends, of course, on experimental confirmation. The efficient operation of the shot damper needs optimal clearance adjustment and, of course, knowledge of it. In order to assess this clearance let us consider the following.

For efficient work we need two impacts per period of vibration, so relative shot velocity (v_r) multiplied by half a vibration period should be equal to the clearance (d), thus

$$v_r \frac{T}{2} = v_r \frac{\pi}{\omega_0} = d. \tag{15}$$

In addition we may assume that the relative velocity is not less than the velocity of the main system:

$$v_r \geq \omega_0 A(\mu \neq 0).$$

So taking into account the equation sign for further consideration and including the efficiency just defined one can obtain

$$d = \omega_0 A(\mu\neq 0)\frac{\pi}{\omega_0} = \frac{\pi A_r(\mu=0)}{E}. \tag{16}$$

And finally, the dimensionless clearance assessment is obtained:

$$D \equiv \frac{d}{A_r(\mu=0)} = \frac{\pi}{E} = \frac{\pi}{1+\frac{2}{\pi}\frac{\mu}{\eta}(1+2\alpha_f)}. \tag{17}$$

So the dimensionless clearance ratio will be of the range

$$D = \begin{cases} \pi \quad \text{for } \frac{\mu}{\eta} \approx 0 \\ \\ 0.5 \text{ for } \frac{\mu}{\eta} = 5, \qquad \alpha_f = 0.5. \end{cases} \tag{18}$$

That means it is of the order of three times the resonant amplitude for very low damper parameter values

$$(\frac{\mu}{\eta} \approx 0)$$

and of the order of one resonant amplitude or less for larger damper parameter values.

4. Experimental verification

In order to verify the findings obtained above, based on the very simple continuous mass and energy approach to the shot damper, a model laboratory experiment was designed and performed. The scheme of the experimental system is shown in Fig. 2, where a cantilever beam is linked with an exciter by a very soft spring and the shot damper is mounted at the top of the cantilever.

There were two versions of this stand, differing only as regards resonant frequencies and loss factors, within the range

$$f_0 \frac{\omega_0}{2\pi} = 8 \div 10\,\text{Hz}$$

and $\eta = 0.0015 - 0.01$. The cantilever end amplitude without the shot mass ($\mu = 0$) during the experiment was of the order $A_r = 10 \div 30$ mm, kept constant, of course, during each series of measurements. Iron balls, steel balls and lead balls of an average radius 2 mm

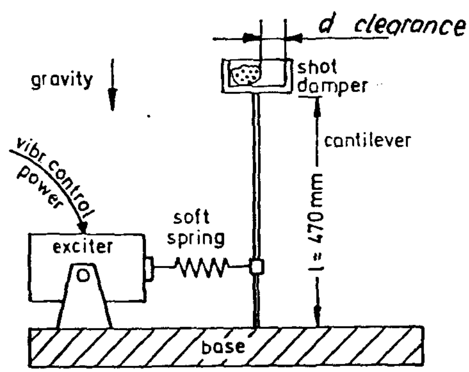

Fig. 2:
Shot damper stand for resonant vibration reduction investigation

were used as shot. The shot mass used in the experiment ranges between 0.5 g and 50 g, while the reduced mass of stand $M = M_{damper} + M_{beam}$ (for the first mode of vibration) was $M = 331$ g and 338 g in the second case. The container walls, originally manufactured from duraluminium, were also alternatively treated with soft plastic and a cardboard. Altogether this method of wall treatment provides three restitution coefficients $R \approx 0.7; 0.5; 0.15$.

The whole experiment performed only at the resonant frequency depends on a stepped clearance adjustment (d) for a given shot mass (m), while looking for a maximum value of vibration reduction efficiency.

In the first version with

$$f_0 = 9.24\ \text{Hz}, \quad \eta = 0.014, \quad M = 331\ \text{g}, \quad m = 3 \div 50\ \text{g},$$

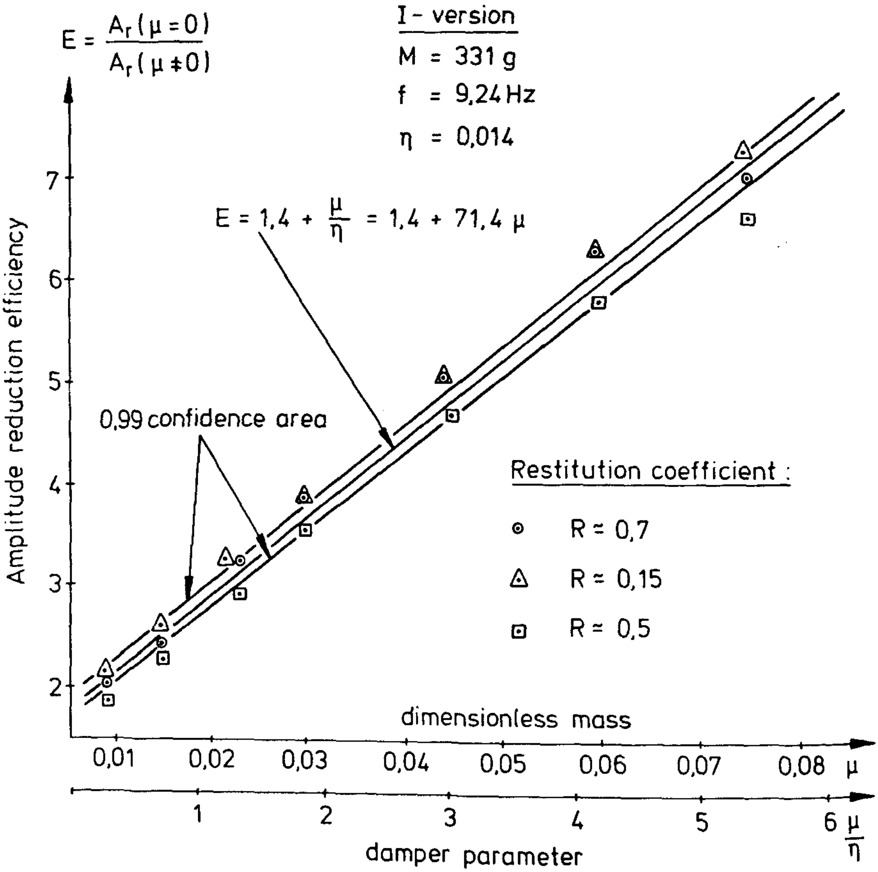

Fig. 3: Elaborated results of the experiment on amplitude reduction efficiency of the shot damper [14].

loose iron shot was used. The results processed statistically and with curve fitting algorithms are shown in Fig. 3 and 4, [14].
The vibration reduction efficiency of the investigated shot damper is shown in Fig. 3 against the damper dimensionless mass

$$\mu = \frac{m}{M} \text{ and also aginst the damper characteristic ratio } \frac{\mu}{\eta}$$

calculated for a given case. As can be seen from Fig. 3, the estimated regression line lies between the lines found theoretically by (14), because outside the

$$\frac{\mu}{\eta} = 0$$

region it is between

$$1 + \frac{2}{\pi}\frac{\mu}{\eta} < E_{exp} = 1{,}4 + \frac{\mu}{\eta} < 1 + \frac{4}{\pi}\frac{\mu}{\eta}. \tag{19}$$

One may thus infer that, in addition to good damping by impact interaction, there was

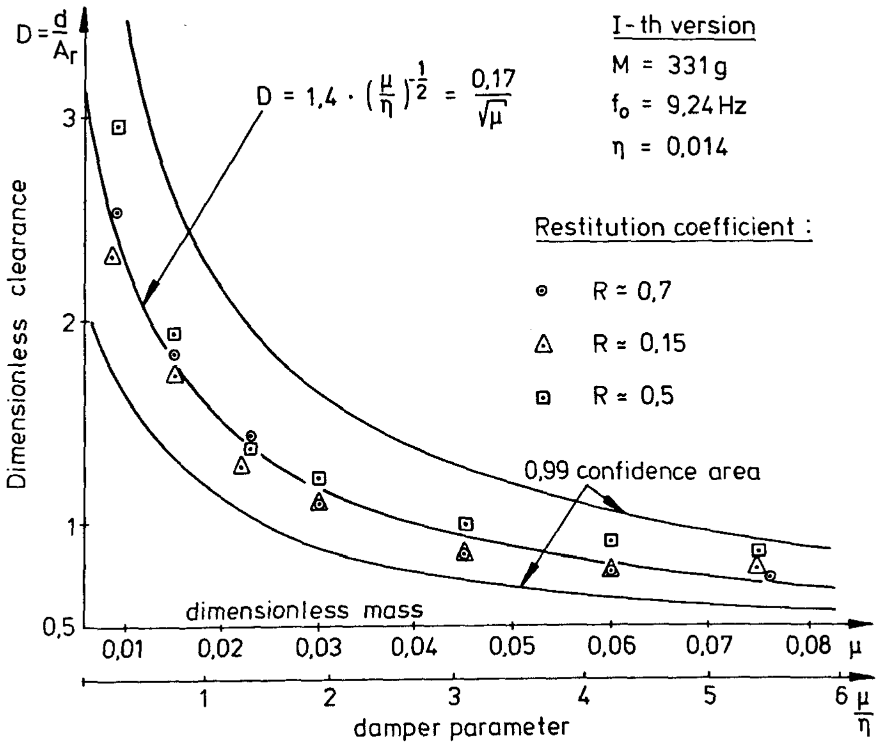

Fig. 4:
Dimensionless clearance of the shot damper as results of the experiment [14].

some friction on the bottom of the container, which greatly improved the shot damper efficiency. One can also see that the restitution coefficient R is not so important here, although the lower it is, the higher is the efficiency value it gives. Thus it provides further confirmation of our theoretical findings.

As far as the clearance results obtained in the first version of the experiment are concerned, they are a little lower than predicted by the theory. But here the theory is much closer to the 99% confidence area of the D $(\frac{\mu}{\eta})$ curve.

Again the restitution coefficient is not the governing factor here, but the lower it is, the smaller will be the clearance, as was predicted by our simple theory.

Based on these introductory results, a similar stand was prepared with greatly improved properties. In particular, the system initial damping was ten times less: $\eta = 0.001469$, container walls much more smooth, the eigenfrequency $f_0 = 8.49$ Hz and almost the same reduced mass $M = 338$ g. Following our theoretical findings, special portions of shot mass

$$\frac{\mu}{\eta} = 1; 2; 3; 4; 5; 6; 7; 8; 12; 16; 22$$

were prepared where the mass increment was $\Delta m = \eta M \cong 0.5$ g at the beginning of the test series.

Many experiments were made, concerning the efficiency increase of the shot damper and also to find the optimal clearance in each case. The friction on the bottom of the container was also increased by grinding. These findings are summarized below (see also Fig. 5):

1. Iron, steel and lead shot of ca. 2 mm diameter was investigated *and lead was found to be the best,* confirming our hypothesis that high efficiency needs $R \approx 0$.
2. Loosely packed lead shot was found to be the optimum method of container filling. This is because shot packed with a plastic cover exhibits the smallest restitution coefficient for the particles used. Thus plastic bags with lead shot give the highest efficiency.
3. One bag with a prescribed amount of shot packed was found to give greater efficiency than the same amount of shot divided into several of the smallest bags.
4. Loose shot or bag, if not guided by special guideways, exhibits parasitic lateral motion without impacts, which lowers the efficiency significantly. This is because the effective mass of the shot is lowered as some part of the total shot mass does not take part in the prescribed one directional motion. It may even reduce the efficiency several times.
5. The highest efficiency can be observed with one plastic bag filled with lead and moving in on the grinded or plastic guideways. In both these cases even for

$$\frac{m}{M} = 0.003 \ (\frac{\mu}{\eta} = 2)$$

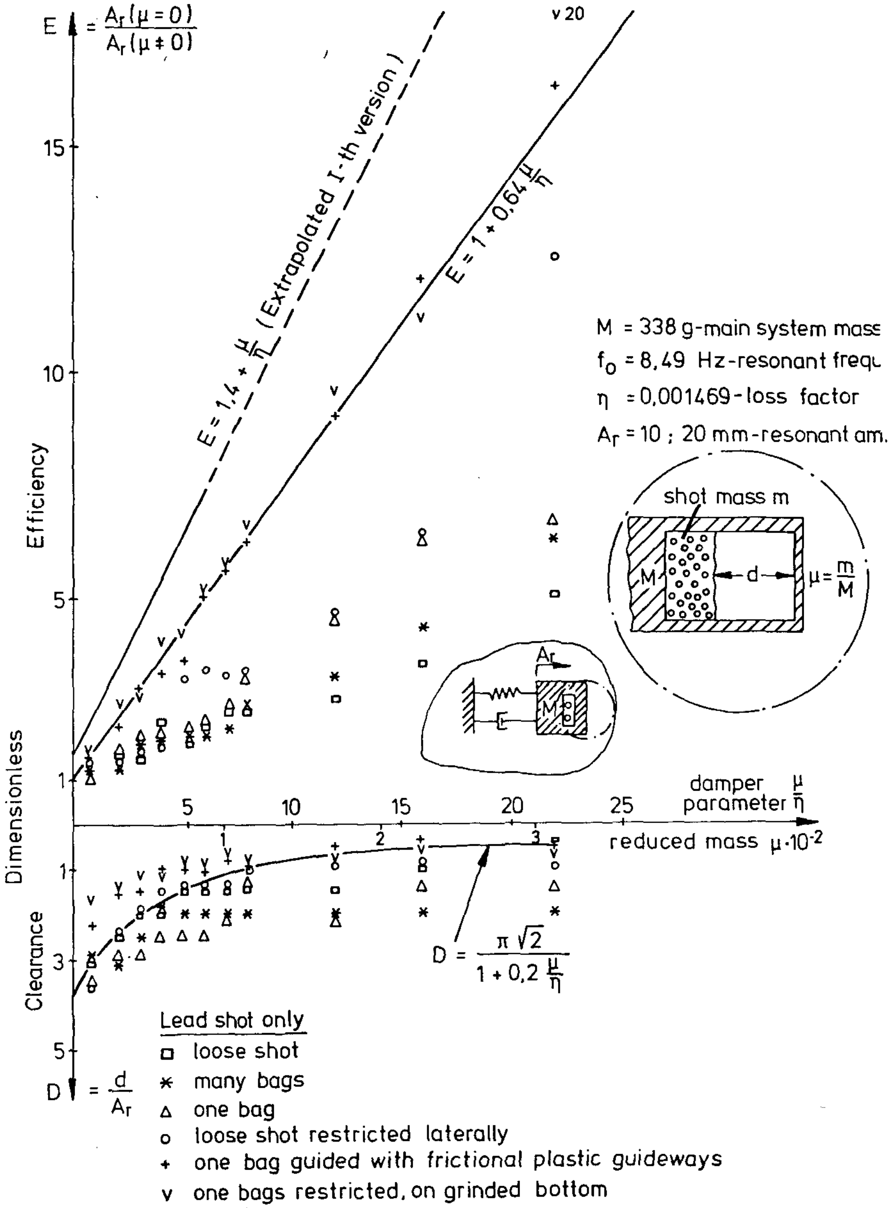

Fig. 5:
Summarized results of a second version of the experiment for the shot damper.

the amplitude reduction of the order E = 2 was observed. Here the efficiency obtained lies between the previously found theoretical lines (14) in the same way as (19), so

$$1+0.64\,\frac{\mu}{\eta} < E_{exp} < 1+1.28\,\frac{\mu}{\eta}. \tag{20}$$

6. The optimal dimensionless clearance D is an average hyperbolic decreasing function. What is more important here is that the higher the efficiency, the lower the clearance. Although the optimal range is between $D = 0.5 \div 2$ which means $d \cong (0.5 \div 2) \times$ resonant amplitude without the shot damper.
7. The value of the dimensionless clearance assessed by formula (17) is in good agreement with the findings, qualitatively and even quantitatively within the average.

So for low friction cases

$$D = \frac{\pi\sqrt{2}}{1+\frac{2}{\pi}\frac{\mu}{\eta}}, \qquad \frac{\mu}{\eta} < 10,$$

and for the higher damper parameter value

$$D = \frac{\pi\sqrt{2}}{1+0.2\,\frac{\mu}{\eta}}, \qquad \frac{\mu}{\eta} > 10 \tag{21}$$

may be a good estimation for the clearance.

So from (17) and the experiment we can draw the important conclusion for application:

$$D \cdot E \sim \pi.$$

All the conclusions listed here may be drawn from Fig. 5, which summarizes our experimental findings.

As is seen from the figure, the condition of the shot motion can improve or destroy the efficiency of the shot damper. Thus in the real case much attention should be paid to ensure the optimal condition of motion in order to obtain the highest efficiency of vibration reduction.

5. Application – shot damper design rule

As was stated in the introduction to this paper, we are looking for a simple design rule for a shot vibration damper. From the measurements or calculations [15] of a real structure we finally know the following data:

1. The vibration amplitude at the point of maximal structure vibration which needs reduction, it is our resonant amplitude – A_r.
2. The loss factor η of a resonance which is the subject of our considerations.
3. The reduced mass of the structure calculated or measured for the given mode of vibration (resonance) – M.

4. The desired amount of vibration reduction – E.

But in order to determine the parameters of the shot damper, i.e. the mass of the shot (m) and the dimensional clearance (d) one has to use two formulas

$$\text{\textit{Efficiency:}}\quad E = 1 + \frac{2}{\pi}\frac{\mu}{\eta}$$
$$\text{\textit{Clearance:}}\quad \frac{d}{A_r} = D = \frac{\sqrt{2}\pi}{E}. \tag{22}$$

Using known data one can thus calculate

$$m = \frac{\pi\eta}{2}(E-1)\,M \text{ – the mass of the shot}$$

and

$$d \approx \frac{\sqrt{2}\,\pi\,A_r}{E} \text{ – rough clearance assessment for further tuning.}$$

As can be seen, the formula for the low friction case is used for the safety case. For a carefully designed and manufactured damper the amplitudes of vibration will therefore not be higher than those calculated. One can also see that the optimal clearance is only assessed with respect to the amplitude range. It has to be adjusted optimally during the initial operation stage. But as is seen from Fig. 5, it is usually of the magnitude of one resonant amplitude A_r value. This problem needs of course further studies.

6. Conclusions

The studies of the efficiency of the shot damper performed here both analytically and experimentally allow one to draw the following conclusions:

1. The efficiency of vibration reduction depends on the damper parameter

   $\frac{\mu}{\eta}$ introduced here.

   This is the ratio of the damper dimensionless mass and the loss factor of the main system. The higher this ratio, the higher will be the efficiency of the vibration reduction.

2. The energy method applied here for the theoretical analysis, together with the "continuous" mass approach, seems to be effective, as was confirmed by experiment. It was thus possible to propose a shot damper design rule for further applications.

3. The energy approach also allows one to discover that the introduction to the damper shot mass of a value μ is equivalent to the external damping introduction of the value

   $$\frac{2}{\pi}\mu \quad \text{or} \quad \frac{4}{\pi}\mu$$

   and higher, if shot friction conditions are optimal. This confirms analytically the experimental findings of the Araki paper [11].

4. The parasite shot lateral motion and also the amount of friction is of great importance. The first has to be limited as much as possible and the second has to have optimal value. Further studies of these two problems are needed.

Summary

The problem of the operation and possible application of the shot damper used for reducing the resonance vibration of mechanical structures is considered. For the theoretical analysis of the shot damper efficiency, the energy method and continuous mass approach was applied successfully. The main findings of this theoretical analysis were confirmed by laboratory experiment. It was thus possible to obtain the design rule for shot damper applications. It was found that the efficiency of vibration reduction depends on the ratio of the damper dimensionless mass μ to the loss factor of the structure η. In addition, the great sensitivity of the efficiency to the path of the shot particle motion and the amount of friction were determined analytically and experimentally.

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